A New Quantized Input RLS, QI-RLS, Algorithm

A. Amiri[1,2], M. Fathy[2], M. Amintoosi[2,3], and H. Sadoghi[3]

[1] Islamic Azad University-Zanjan Branch
[2] Faculty of Computer Engineering, Iran University of Science and Technology
[3] Faculty of Engineering, Tarbiat Moallem University of Sabzevar, Iran
a_Amiri@comp.iust.ac.ir, mahFathy@iust.ac.ir,
mAmintoosi@iust.ac.ir

Abstract. Several modified RLS algorithms are studied in order to improve the rate of convergence, increase the tracking performance and reduce the computational cost of the regular RLS algorithm. . In this paper a new quantized input RLS, QI-RLS algorithm is introduced. The proposed algorithm is a modification of an existing method, namely, CRLS, and uses a new quantization function for clipping the input signal. We showed mathematically the convergence of the QI-RLS filter weights to the optimum Wiener filter weights. Also, we proved that the proposed algorithm has better tracking than the conventional RLS algorithm. We discuss the conditions which one have to consider so that he can get better performance of QI-RLS against the CRLS and standard RLS algorithms. The results of simulations confirm the presented analysis.

Keywords: Adaptive Filter, Recursive Least Square (RLS), Weiner Optimum Weights, Tracking.

1 Introduction

An adaptive filter is a filter which self-adjusts its transfer function according to an optimizing algorithm. Because of the complexity of the optimizing algorithms, most adaptive filters are digital filters that perform digital signal processing and adapt their performance based on the input signal. The Recursive Least Square (RLS) and the Least Mean Square (LMS) are two famous adaptive filtering algorithms [5]. They have attained its popularity due to a broad range of useful applications in such diverse areas as communications, radar, sonar, seismology, navigation and control systems, and biomedical electronics.

The optimization of convergence speed and tracking performance are open problems in adaptive filter theory. Fast convergence of the RLS has given rise to the development of the algorithms based on it [6,9,10,12].

The works reported in [1,2,3,7,8,14] have been done for increasing the real-time performance of the LMS algorithm using the sign of the input data and/or error during updating the filter weights. In the clipped RLS algorithm [11], the input signal is quantized into three levels of -1, 0, +1. They discussed the convergence and the computational complexity of their own CRLS algorithm.

O. Gervasi and M. Gavrilova (Eds.): ICCSA 2007, LNCS 4707, pp. 495 – 506, 2007.

In the three levels clipping method the small domain signal is assumed as noise and it is obvious that it causes a lot of lost in input information in signals with low SNR[1]. In the proposed new clipped RLS, QI-RLS, algorithm the input has been clipped, such that we save more information and in the same time, the convergence and tracking performance of the proposed algorithm, are remained. The mathematical proofs and simulation results shows the better performance of the proposed method against Standard RLS and CRLS.

The variants of RLS are discussed in Section 2. The proposed new algorithm, which is a modification of the aforementioned algorithm, appears in Section 3. Section 4 deals with computer simulation issues. The final section presents conclusion and summarizes the main findings.

2 RLS Algorithm

In this section we review the Standard RLS and the CRLS Algorithm [11] which is the foundation of our proposed algorithm.

2.1 Standard RLS Algorithm

RLS algorithm has been explained in many literatures such as [5, 13]. In this section we review briefly this algorithm. The RLS predictor algorithm has been studied in [11] as:

$$W_{n+1} = W_n + R_n^{-1} X_n e_n \tag{1}$$

Where:

$$R_n^{-1} = \lambda R_{n-1}^{-1} - \frac{\lambda^{-2} R_{n-1}^{-1} X_n X_n^T (R_{n-1}^{-1})^T}{1 + \lambda^{-1} X_n^T R_{n-1}^{-1} X_n} \tag{2}$$

$$e_k = d_k - X_k^T W_k \tag{3}$$

$W_k = [w_k(1), w_k(2), ..., w_k(N)]^T$ is the weight vector of the predictor, $X_k = [x_k(1), x_k(2), ..., x_k(N)]$ is the vector of the input data sequence, which is assumed to be a stationary random process, *N* is the number of filter tapes, e_k is the estimation error and d_k is the desired response.

2.2 Clipped RLS Algorithm

Sadoghi et al. [11] have proposed Clipped-RLS as a variation of the standard RLS algorithm. They quantized the input signal with msgn(.) into a three-level signal. The msgn(.) function is described in equation 4 and is shown in Figure 1:

[1] Signal to Noise Ratio.

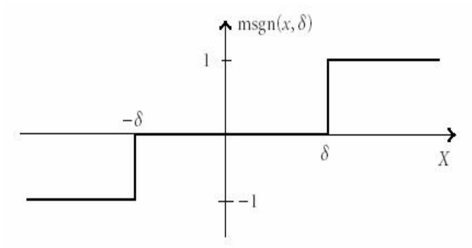

Fig. 1. Quantization scheme for the CRLS algorithm proposed in[11]

$$m\mathrm{sgn}(x,\delta)=\begin{cases}1 & x>\delta\\ 0 & -\delta\le x\le\delta\\ -1 & x<\delta\end{cases} \tag{4}$$

According to msgn(.), the estimated input signal, $\hat{X}_n$ was replaced with X_n in Equations (2), (3). They also discussed the convergence and the computational complexity of their own CRLS algorithm.

In the next section we will explain the proposed new Quantized Input RLS (QI-RLS) and will discuss its performance, convergence and tracking.

3 The Proposed QI-RLS Algorithm

As we have seen in the previous section, the CRLS filter quantizes the input signal to three levels (Figure 1). Although it has been mentioned in [11] that it reduces the noise effect, but it is obvious that for inputs in range $[-\delta,+\delta]$, we have a lot of lost in input information. Because for that range, whole input assumed as noise. In the proposed new quantized input RLS, QI-RLS algorithm the input has been clipped such that we save all of the information for inputs in range $[-\delta,+\delta]$. Equation 4 shows our new clipping function that we named it $tgn(x,\delta)$ and is demonstrated in figure 2:

$$tgn(x,\delta)=\begin{cases}1 & x>\delta\\ x & -\delta\le x\le\delta\\ -1 & x<\delta\end{cases} \tag{5}$$

The adaptation equation can be written as:

$$W_{n+1}=W_n+R_n^{-1}X_n e_n \tag{6}$$

$$R_n^{-1}=\lambda R_{n-1}^{-1}-\frac{\lambda^{-2}R_{n-1}^{-1}\hat{X}_n\hat{X}_n^T(R_{n-1}^{-1})^T}{1+\lambda^{-1}\hat{X}_n^T R_{n-1}^{-1}\hat{X}_n} \tag{7}$$

$$e_k=d_k-X_k^T W_k \tag{8}$$